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Optimum ship routing for the north Indian Ocean region – a decision support system

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ABSTRACT

This paper presents a general approach for the development of a decision support system (DSS) for ship weather routing. With advances in the weather and climate modelling coupled with availability of large computing power, it has become increasingly common to base the route decisions on weather and in particular wave forecasts. The term optimal means a route that optimizes any one or a combination of the factors such as maximum safety and crew comfort, minimum fuel consumption, minimum transit time etc., depending on the vessel, voyage type and mission requirements. Djikstra's path optimization scheme, which employs optimal control theory and dynamic programming technique, is used to obtain optimum route in a given random sea-state. The developed algorithm is investigated using various realistic wave data for the North Indian Ocean region obtained from a 3rd generation WAM model. Illustrative examples of minimum-time sea routes on Arabian Sea and Bay-of-Bengal have been determined and presented. All relevant practical and realistic constraints such as presence of land boundaries, consideration of non-navigable water, effects of wind and current, voluntary speed reduction etc. are incorporated and discussed within the framework of the algorithm.

Keywords: Ship routing, Wave modelling, Sea keeping, Added Resistance, Route Optimization.

1. INTRODUCTION

Ship routing is an 'art of science' to determine an optimal track for ocean voyages based on forecasts of weather, sea conditions, and a ship's individual characteristics for a particular transit. There are a number of different approaches to calculate the shortest path/route between two nodes representing the start and destination ports. For a given problem, the implementation of this path finding algorithm may have to be executed several times due to the varying ocean wave conditions during the passage of the ship. Within specified limits of weather and sea conditions, the term 'optimum' may be defined to mean maximum safety and crew comfort, minimum fuel consumption, minimum time underway, or any combination of these factors. Development of optimal ship-weather-routing requires a combination of the following three different areas:

a. Forecasting the sea-conditions. (i.e. ocean-state forecast)
b. Estimating ship behaviour in such ocean wave conditions.
c. Development of an appropriate and efficient track or path optimization algorithm.

The last aspect above, i.e. a 'track-optimization' algorithm wherein some objective function is optimized (maximized/minimized) combines the first two areas. The problem of obtaining an optimal trajectory of ship has attracted attention of many researchers in the past. According to Hanssen and James (1960), an optimum ship route was developed under stationary weather conditions. Development of more realistic routing mechanisms using variational methods assuming ship speed under maximum power to be independent of time was attempted by Haltiner et al. (1962). Models based on numerical methods to propose an optimal trajectory was attempted by Faulkner (1964). These models however, had constraints pertaining to undesired ship motions and treatment of continental obstacles. Zoppoli (1972) formulated that the minimal time algorithm as an N-stage discrete process subjected to stochastic and dynamic conditions. Chen (1978) developed an adaptive open-loop feedback optimization procedure, where a ship is deterministically rerouted when new information shows that environmental conditions are sufficiently different from the initial estimates. Mitchell and Papadimitriou (1986) investigated the shortest path through a weighted planar sub-division. Hagiwara (1985, 1989) proposed the isochrone method for the solution of the minimum-time route (MTR) problem. Earlier researches and more recent works from Perakis, A. and Papadakis (1989), Chen, H. and Lacey (1998), Ulusoy (2004), Montes (2005), Christiansen(2013), Gonzalo(2013), Lin (2014) focus on the optimization algorithm using advance mathematical models, their use for developing an operational algorithm for practical application are not often demonstrated and thus their actual application remains somewhat unclear. From the above it can be seen that although ship weather routing is a fairly old problem, due to its commercial value, the exact operational algorithm is rarely available in open literature. In this paper, taking advantage of the modern developments in wave modelling, an optimum track ship routing algorithm for ships operating in the Indian Ocean region is described by using a network graph for the Indian Ocean.

2. ARCHITECTURAL FRAMEWORK OF ROUTING ALGORITHM

Wave Climate and Ship Responses in Waves

The most important environmental factors relating to ship's safety and performance on the high seas are surface winds and waves. For the wave climate, the final input that is needed for evaluating the vessel behaviour is the spectral representation of the waves. Significant advance have been made over the past few decades in both quality and quantity of satellite-generated wave data as well as in the ocean-wave prediction models. To determine ship behaviour in a realistic wave field, the usual procedure is to use a theoretical wave spectrum. The mathematical formulations of these normalized uni-directional (i.e. long-crested or two-dimensional) wave energy spectra are based on one or two parameters: the significant wave height, and/or average wave periods. In the present work, as it is usual for ship applications, we apply the ITTC spectrum based on the 3rd generation WAM generated data on significant wave height and representative periods. This means over the applicable ocean region WAM (3rd-G Wave Model) produces the mentioned wave parameters which are then used with the relevant theoretical ITTC formula to generate the wave energy spectrum. The method to determine ship behaviour in irregular waves defined by means of a wave spectrum, using the response parameters in regular waves or so-called RAO (Response Amplitude Operator) is well known and is available in any standard text of naval architecture (Lewis, 1989). The most important relevant RAO's can be determined by using the fundamental six-modes of motions, and then the well-known Salvesen-Faltinsen-Tuck (1970) version of the strip theory for these computations. As for added resistance, which is a second order force, there are several formulae for its determination in terms of the heave and pitch forces response parameters (Bhattacharyya, 1978 and Faltinsen, 1990). However, in our work we used the formula of Beukelman and Gerritsma (1972).

Ship Route (Track) Optimization

Here, the term 'optimum' means a route that optimizes any one or a combination of the factors such as maximum safety and crew comfort, fuel consumption, transit time etc., depending on the vessel, voyage type and mission requirements. For most transits this will mean the minimum transit time that avoids significant risk to the vessel, crew and cargo. The routing goal may not always be to reduce the time of transit. Sometimes the goal will be to reduce fuel consumption and to keep a vessel on a regular schedule. At present, there are several techniques are available to reduce the fuel consumption, which can be related to a reduction in engine power/speed (objective function) under the following heads:

- Involuntary speed reduction
- Voluntary speed reduction

Involuntary speed reduction is due to the increased resistance in a seaway, while voluntary speed reduction is the deliberate reduction in speed by the ship's captain in order to ensure that the ship's wave-induced responses are within acceptable safety limits, since it is found that in general a reduction in speed (and also heading to some extent) improves sea-keeping (i.e. reduces motions). For example, there may be a maximum limit to the bow slamming, or a maximum acceptable roll angle etc.

The Track Optimization Algorithm

Routing of ships based on wave conditions was being practiced by seafarers and ship navigators since a long time. The optimization algorithm to be used should be general enough so that it can handle variety of multiple objective criteria. In general, the optimal-path problem is a problem of interest in many fields of study, e.g. traffic engineering. A literature search on general path-optimization algorithm reveals that one of the available and easy to implement optimization technique in this context is the Dijkstra's algorithm (Cormen et al., 2001 and Taha, 2002), which tries to minimize the distance between any two node points in a given mesh/grid. The distance can also be replaced by any 'weight' function. In the case of the ship routing problem, the weights can be viewed as an 'objective' or 'achievement' function, which can be obtained by combining weather information along with sea-keeping characteristics of the hull. It was thus felt that this algorithm could be successfully applied for the present ship-routing problem.

Determination of 'weights'

In order to apply Dijkstra's algorithm for the ship routing problem, an area of the sea-surface encompassing the possible route of the ship need to be discretized by means of a grid formed by latitudes and longitudes. The nodes or vertices can be taken as a central point in the grid or alternatively the intersection of each latitude and longitude line can be considered as a node. Typically the weather information (significant wave height and characteristic wave period) will be available from satellite generated data and advance wave modeling methods like WAM at each grid. As in WAM output, these values are assumed constant over the corresponding grid. The next and most important task is now to determine an appropriated 'weight' function for each grid or node by combining the wave conditions (significant wave height and direction). Once these 'weights' are known at each node/grid, the problem is now transformed to a state where Dijkstra's algorithm can be applied to find the 'optimal' path for minimum (or maximum) 'weight'. The weights $w_{i,j}$ between the path lines joining adjacent nodes i, j will depend on the parameter that need to be optimized. There can be several possible parameters such as minimum travel time, minimum fuel consumption, safe and comfortable travel etc. In the present work, we considered the optimum ship routing for minimum travel time and discuss the methods to determine the weight function based on involuntary speed reduction. For this we first need to determine the reduction in speed due to the added drag. In calm water, the effective power P_E is given by $R_{SW}(V)V$, but in the presence of waves, winds and current, the effective power will be given by $R_T(V_R)V_R = (R_{SW}(V_R) + R_{add}(V_R))V_R$. Here R_{add} is the resistance due to winds, waves and current, i.e. $R_{add} = \overline{R}_{AW} + R_W + R_C$. This is illustrated in figure 2. In order to determine the reduced speed from this, we note that the calm water resistance can be expressed in the form aV^2 where $a = 0.5\rho SC_{SW}$. As regards the additional drag, if an assumption is made that the power required for this component at speed V_R is same as the power required at speed V, i.e. AB = CD in figure 1, then we get,

$$aV_R^{\,2}V_R + R_{add}(V)V = aV^2V \qquad (1)$$

From above, we can get the reduced speed as:

$$V_R = \left(V^3 - \frac{R_{add}(V)V}{a} \right)^{1/3} \qquad (2)$$

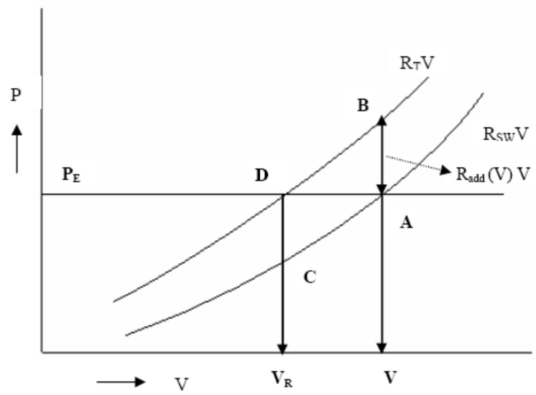


Figure 1 Procedure to obtain reduced speed for constant effective power

In order to simplify, we now made an ad-hoc assumption that the effective thrust, i.e. $(1-t)T$ does not vary much over the range V_R to V, and therefore we can take it as constant as far as determination of reduced speed V_R is concerned. We note that the procedure to determine the total added resistance are all based on approximate theoretical and/or semi-empirical formulations, and therefore there is quite some uncertainty in these values. Additionally, there will also be some uncertainty in the forecast weather information like wave data and wind data. Further, it also needs to be noted that prevailing weather conditions are all assumed constant over a grid, which itself is an averaging process introducing some inaccuracy. It also needs to be noted that the error introduced in assuming thrust to remain constant will have similar order of error for all possible paths, and therefore the determined optimal path will still remain optimal or near optimal. Thus determination of the reduced speed based on the assumption that effective thrust is constant over the small range of speed around the prevailing calm water speed may be acceptable for practical calculations.

Based on the above assumption, the reduced speed will be as depicted in figure 2. If we write

$R_{add}(V)/R_{SW}(V) = k$,

then,

$$R_T(V) = R_{SW}(V) + R_{add}(V) = R_{SW}(V)(1+k) \qquad (3)$$

If a database is created by computing $R_T(V)$ against V for different values of k, then for any given k, one can determine the corresponding speed V. Hence by a process of interpolation, for a given additional resistance value, one can determine the reduced speed V_R, as shown in figure 2. This procedure is however based on expressing the additional resistance as $k\,R_{SW}$. This means, the additional resistance due to environmental factors are all taken as a function of the calm water resistance regardless of speed, which means it has the implicit assumption that the speed dependence of R_{add} is same as that of R_{SW}. Instead of this, it may be more reasonable to assume that $R_{add}(V_R) = R_{add}(V)$, as illustrated in figure 2. In such a case, if we have R_{SW} expressed as aV^2, we can directly get the reduced speed as:

$$V_R = \left(V^2 - \frac{R_{add}(V)}{a} \right)^{1/2} \qquad (4)$$

Both the approximate procedures given by (2) and (4) will require determination of the additional resistance R_{add} arising from wind, waves and currents for each possible path line between adjacent grids using the prevailing environmental conditions at that

location, but for the given calm water ship speed V. Once the reduced speed for each possible path line joining nodes i and j is determined, i.e. $V_{R_{i,j}}$ is found, it is straightforward to determine the weight functions as the time $t_{i,j}$ taken to cover the distance between the nodes i and i. If this distance is $L_{i,j}$, we have:

$$w_{i,j} \equiv t_{i,j} = \frac{L_{i,j}}{V_{R_{i,j}}} \tag{5}$$

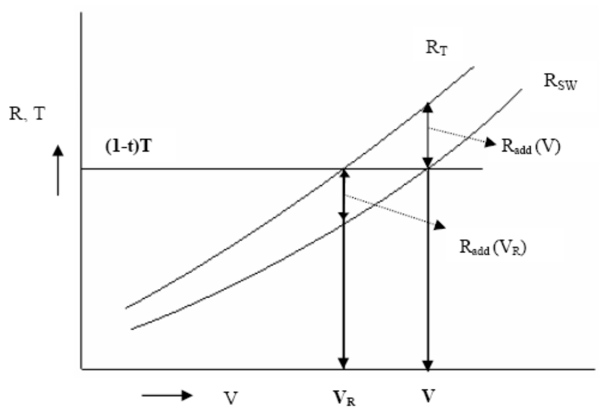


Figure 2 Procedure to get reduced speed assuming constant thrust

By optimizing (minimizing) $w_{i,j}$ for the complete path, the minimum time travel route for the ship can now be found. Here the route optimization due only for involuntary speed reduction, that is due to additional drag arising from environmental factors have been considered. It may be noted that in case of calm water, $w_{(i,j)}$ reduces to $t_0(i, j) = L_{i,j} / V$, which is the travel time taken at constant calm water resistance, and the minimum-time route becomes the shortest route. In the present work, both (2) and (4) are used, and it is found that the difference in the predicted path from these two is marginal. Therefore finally the approximate formula (4) is retained, as it is found to be the most convenient and easy to apply.

3. RESULTS AND DISCUSSION

A large number of results have been previously published for demonstrating the working of the algorithm, and its correctness in achieving the minimal time route [Padhy, 2011 & Padhy et al., 2008,2009,2011,2015], and thus these are not repeated here. Here we apply the algorithm to determine minimum-time optimal route for two ships termed Ship1 and Ship2 for routes lying in the North Indian Ocean region, for which realistic wave data are generated using the 3rd generation WAM. The wave model is run using the NCMRWF (National Centre for Medium range Wave forecast, at Delhi) wind field data. Ship1 is a relatively small vessel of length 60m, breadth 11m and draft 2.9m, while Ship2 is of length 160.93m, breadth 23.1m and draft 9.07m. Figure 3 shows the route between Damman and Mumbai, and the return route of Mumbai-Damman for Ship1, while for the same routes for Ship2, results are shown in figure 4. The comparison of the results for these two ships is depicted in figure 5. These results show that the routes depend on the ship type, and also on their direction with respect to the waves.

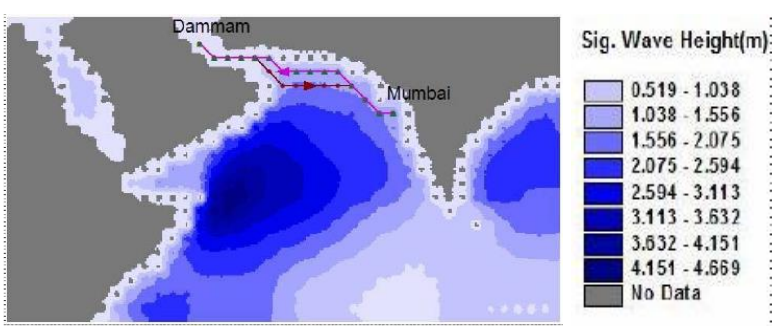


Figure 3 Route between Dammam - Mumbai and Mumbai - Dammam by Ship1

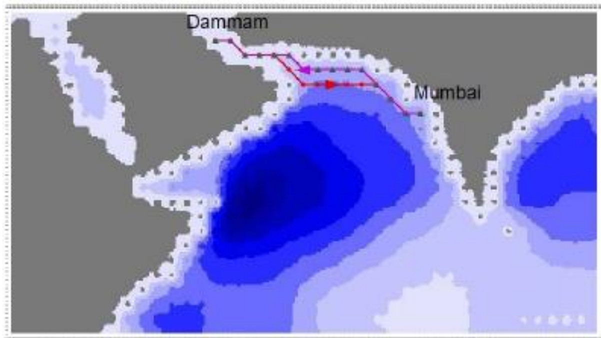


Figure 4 Route between Dammam - Mumbai and Mumbai - Dammam by Ship1

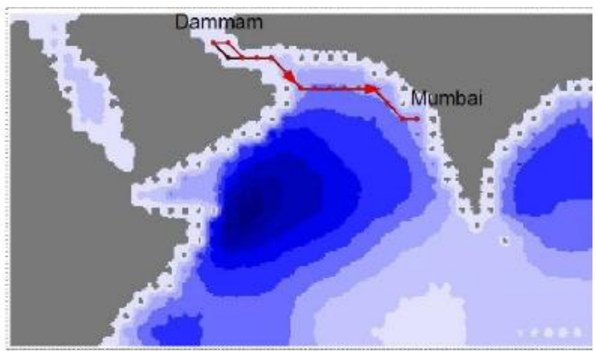
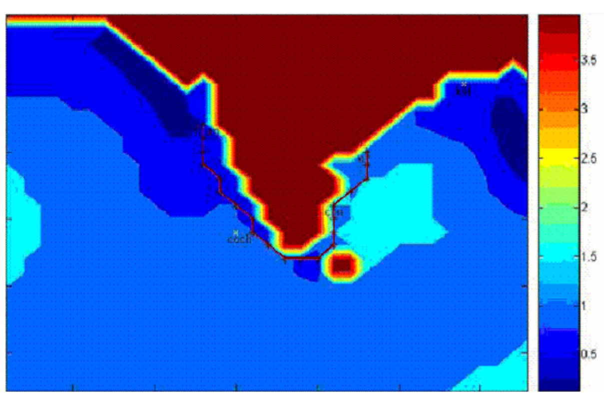


Figure 5 Comparisons between Ship1 and Ship2 routes



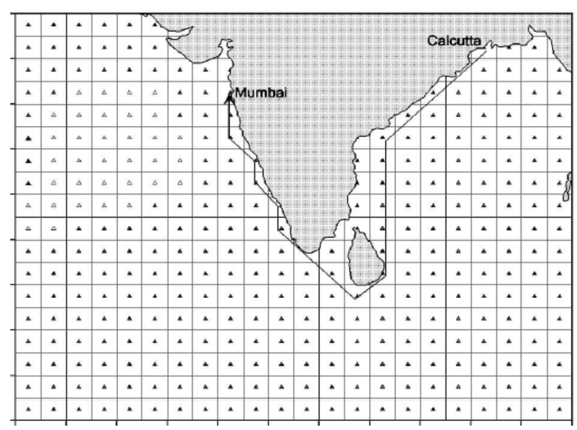


Figure 6 Route between Mumbai-Visakhapatnam

Figure 7 Kolkata-Mumbai route avoiding Palk-Strait

In order to demonstrate the ability of the algorithm to generate route circumnavigating a land mass in figure 6, we show the route from Mumbai on the west coast of India to Visakhapatnam on the east coast of India. The result is for Ship1. The algorithm here simply assigns very large wave heights to those grids which are land masses. As can be seen, the algorithm could produce a route going around the land mass. However, there is a problem with this track: this route goes through the Palk Strait, the water between India and Sri- Lanka. This Strait is not open for navigation, and therefore ships are not allowed to go through it. Such constraints are also easily handled by the algorithm, by simply `blocking' selected areas of ocean. This is achieved by simply taking such areas as part of land mass. Thus isolated areas on open Ocean which are not open to navigation can be treated as islands by simply assigning very large wave heights associated with those grids. Result is shown in figure 7.

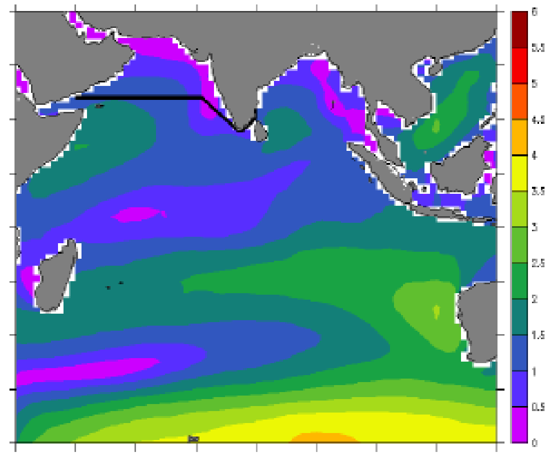


Figure 8 Route between Adan - Chennai based on WAM climatological wave data for January

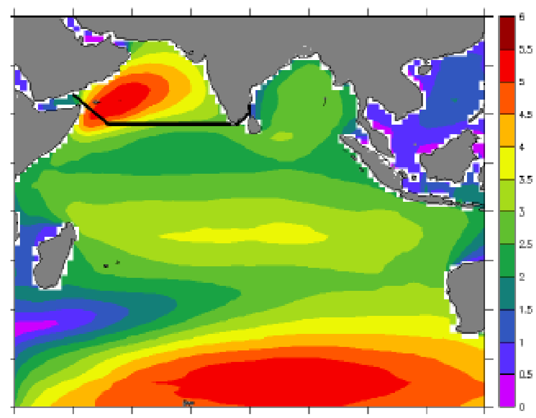


Figure 9 Route between Adan - Chennai based on WAM climatological wave data for July

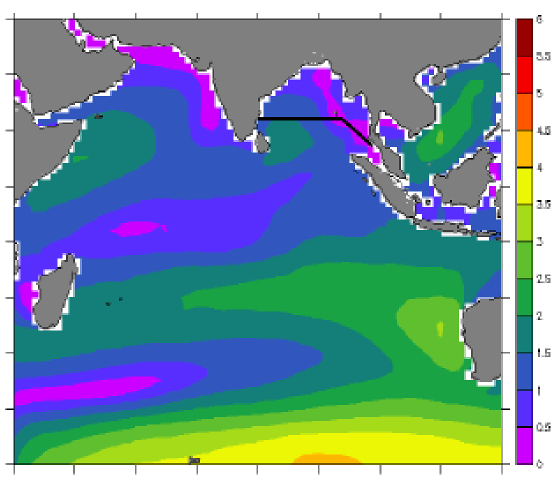


Figure 10 Route between Chennai – George Town based on WAM climatological wave data for January

Figures 8 & 9 show the result from the algorithm where the monthly average wave-data is used. Although the algorithm is run for all 12 months, here for brevity we show results for only two months, January and July, the former being a winter month with relatively calm sea and the latter being a monsoon month with high sea conditions. Note that here Palk strait is treated as navigable

water. As can be clearly seen, the routes generated are quite different. Such studies are therefore of great use for planning routes based on time of the year for the voyage. Similar result for a route lying in Bay of Bengal is shown in figures 10 & 11. These figures show that here the routes between winter and monsoon months are not as large. This is mainly because the region through which the route passes does not show large change in the wave heights, although in other parts of the ocean there are large changes in wave heights.

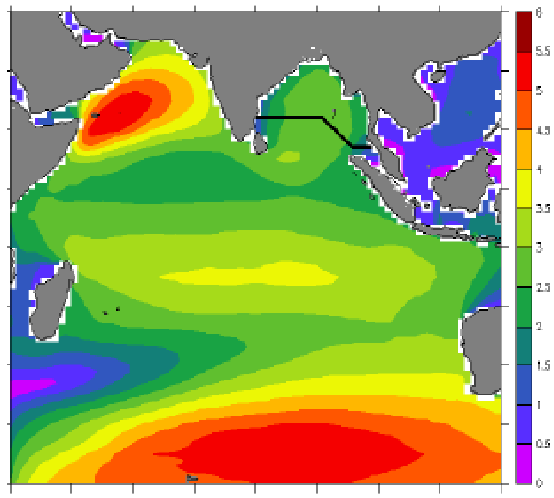


Figure 11 Route between Chennai – George Town based on WAM climatological wave data for July

One problem for this algorithm is that the route produces often shows a zig-zag saw-tooth type nature lacking smoothness, e.g. see figure 6. This is a result of generating the route by adding the nodes at the centre of the grids. A `smoothing-scheme' is therefore devised to handle this problem, and the results of applying this scheme is shown in figures 13, 14. It can be seen that the generated track is now smooth.

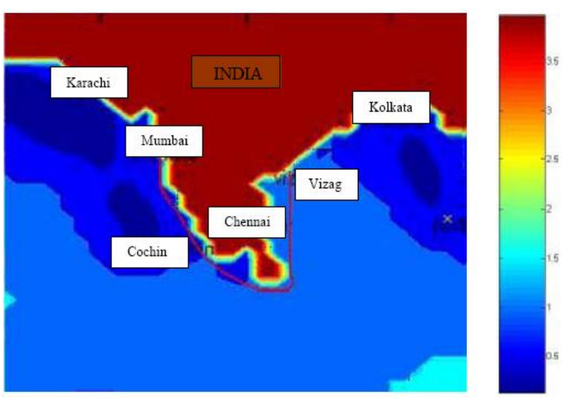


Figure 12 'Smoothed' track between Mumbai and Vizag

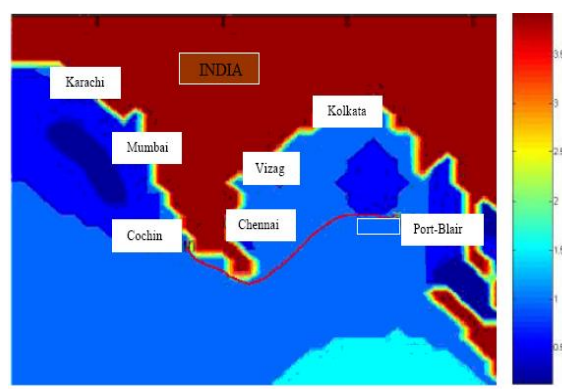


Figure 13 'Smoothed' track between Cochin and Port Blair

The algorithm, as developed is also capable of handling presence of current and wind fields: these can be considered by simply taking the additional drags induced by these effects. Formulations are available for estimating current drag and wind loads. Figure 14 shows a minimum-time route in a wave and current field. In a similar way, the effect of wind drag can also be considered. Finally, the algorithm is extended for considering voluntary speed reduction, in addition to the involuntary speed reduction in planning the route. To account for the voluntary speed reduction within the framework of the developed algorithm, the maximum attainable speed at each grid considering both voluntary and involuntary speed reduction needs to be found. This is schematically shown in figure 15.

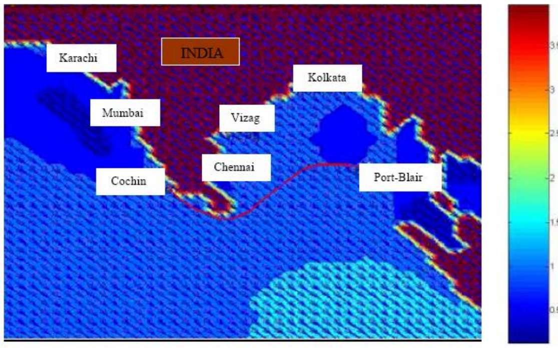


Figure 14 Optimum Cochin - Port Blair route with waves and current

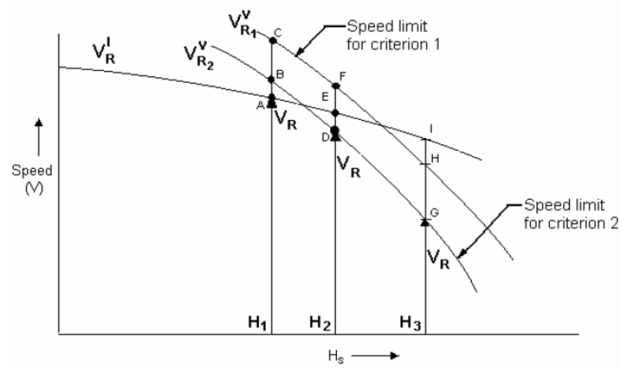


Figure 15 Illustration for consideration of voluntary speed reduction

Let V_R^I represent the reduced speed due to involuntary speed reduction (due to added wave drag), V_{R1}^V and V_{R2}^V represent the maximum speed limits for response criterion 1 and 2 respectively. For wave height H_1, the speeds V_R^I, V_{R2}^V and V_{R1}^V are given by points A, B, C respectively, and since $V_{R1}^V, V_{R2}^V > V_R^I$, the net reduced speed for this height will be V_R^I. For wave height H_2, the corresponding points are respectively D, E, F and since here $V_{R2}^V < V_R^I$ but $V_{R1}^V > V_R^I$, V_{R2}^V, the speed limit here is V_{R2}^V. For height H_2, the points are G, H, I. Here, both $V_{R1}^V, V_{R2}^V < V_R^I$ and also $V_{R1}^V > V_{R2}^V$ and therefore the limiting speed will be V_{R2}^V. In other words, for any wave height, the final reduced speed will be the lowermost of the three speeds V_R^I, V_{R1}^V and V_{R2}^V. Thus in general, we have:

$$V_R = \min(V_R^I, V_{Rm}^V, m = 1, 2, ..., N_R) \qquad (6)$$

where V_R is the reduced speed, and $V_{Ri}^V, i = 1, 2, ..., N_R$ are the maximum speeds based on N_R number of response criteria. Once V_R for a given path line is determined, the weights for the minimum-time route is found as in eqn. (5).

Typical results with and without consideration of voluntary speed reduction for Ship2 are shown in figures 16-18. Figure 16 shows the route without voluntary speed reduction. Figure 17 is the route with voluntary speed reduction based on a limiting set of values of primary roll, heave and pitch motions (max. roll 10 deg., max. heave 1m and max. pitch 3 deg.). Figure 18 shows the plot with voluntary speed reduction, but now with a different set of value of max. roll 5 deg. In these results it is found that voluntary speed reduction has hardly influenced the route except over a small part. This however is because of the moderate sea conditions: in these waves the ship is found to hardly exceed the set criterion of the motions.

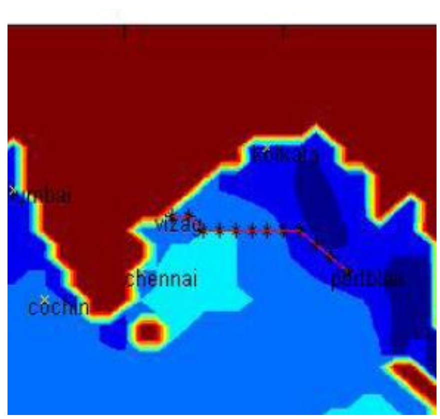


Figure 16 Vizag-Port blair route without voluntary speed reduction

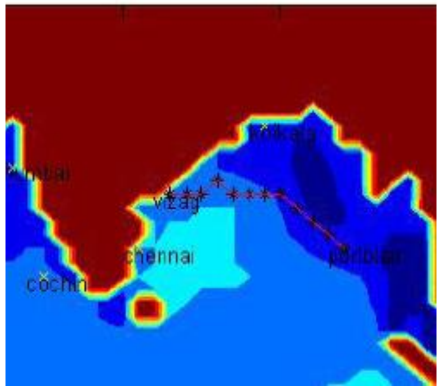


Figure 17 Vizag-Port Blair route with voluntary speed reduction, max roll 10deg, pitch 2deg. and heave 1m

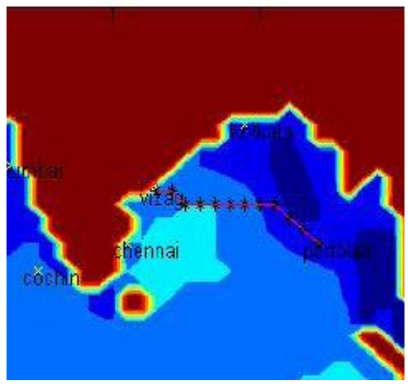


Figure 18 Vizag-Port Blair route with voluntary speed reduction, max roll 5 deg., pitch 2deg. and heave 1m

4. CONCLUSION AND FUTURE WORK

In this work, an algorithm has been developed for ship weather routing application considering the prevailing weather (wind-wave-current) conditions and ship behavior in waves. The specific application in this work has been confined to routes lying in the North Indian Ocean region, primarily because of the availability of information from ISRO satellites and from sites such as NCMRWF of India providing ocean wave related data over this region. The optimization procedure is based on a variant of Dijsktra's algorithm, which has been suitably modified for application to the ship-routing problem. Here, the optimization criterion is chosen to be the time of travel, so that the obtained path represents a 'minimum-time-travel' route. For this, the procedure to determine the appropriate weight functions has been explained. It is found that the algorithm is versatile and robust enough to handle all constraints that are usually present in practical application of ship routing by ship operators. After successful validation of the developed routing routine, a suitable fuel optimization model is under way to integrate into it. Ship-routing has a large commercial application and there are several weather-routing service providers available internationally. However, most of these service providers, do not give much information on the optimization method. In open literature also, information available on the basic method or algorithm that can be used for practical application is scarce. The commercial codes provide no information except the claimed capability of their services, while most research papers tend to deal with theoretical aspects of optimization algorithms whose application for practical ship navigation is generally unclear. To this end, this paper has presented the strategy of a routing scheme towards the competitive advantage, which may be possible to be applied in practice by ship operators.

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